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Robust Algorithm for Discrete Tomography with Gray Value Estimation

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Abstract — *In this paper, we present a novel iterative reconstruction algorithm for discrete tomography (DT) named total variation regularized discrete algebraic reconstruction technique (TVR-DART) with automated gray value estimation. This algorithm is more robust and automated than the original DART algorithm, and is aimed at imaging of objects consisting of only a few different material compositions, each corresponding to a different gray value in the reconstruction. By exploiting two types of prior knowledge of the scanned object simultaneously, TVR-DART solves the discrete reconstruction problem within an optimization framework inspired by compressive sensing to steer the current reconstruction toward a solution with the specified number of discrete gray values. The gray values and the thresholds are estimated as the reconstruction improves through iterations. Extensive experiments from simulated data, experimental μCT, and electron tomography data sets show that TVR-DART is capable of providing more accurate reconstruction than existing algorithms under noisy conditions from a small number of projection images and/or from a small angular range. Furthermore, the new algorithm requires less effort on parameter tuning compared with the original DART algorithm. With TVR-DART, we aim to provide the tomography society with an easy-to-use and robust algorithm for D T .*

I. INTRODUCTION

Tomography is a powerful technique for investigating the three-dimensional (3D) structures of objects by utilizing penetrating waves or particles, and has a wide range of applications such as computed tomography (CT) [1] and electron tomography (ET) [2]. Projection images of an object are acquired over a range of rotation angles, and a mathematical procedure known as tomographic reconstruction is required to recover the 3D object information from its 2D projections. Due to its large influence on the outcome of the complete tomography experiment, reconstruction algorithms have been a subject under intensive research [3], [4].

In most practical applications, it is extremely advantageous if the reconstruction algorithm can still produce accurate results using a small number of projection images under moderate/high noise levels. For example, medical CT uses ionizing radiation and reducing the dose to the patient is of high importance. In transmission electron tomography, the electron beam causes damage to the sample during acquisition and cannot penetrate the sample section at high tilt angles. These practical aspects limit the number of projection images, acquired image quality (low dose leads to high noise level), and/or available angular range for reconstruction. Under such conditions, the tomography reconstruction problem is highly underdetermined and there is no unique solution to the inverse problem based on only the acquire data.

This necessitates the full utilization of the prior knowledge we have on the unknown object. Compressive sensing (CS) is one of the concepts under intensive research in recent years [5], [6]. It proves that if an image is sparse in a certain domain, it can be recovered accurately from a small number of measurements with high probability when the measurements satisfy certain randomization properties [7]. Total Variation Minimization (TVmin) can be seen as a special case of CS when the boundary of the object is sparse within the image [8]. Discrete tomography (DT) considers another type of prior knowledge where the object is known to consist

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of a limited number of materials, each producing a constant gray value in the reconstruction. The Discrete Algebraic Reconstruction Technique (DART) is one of the practical algorithms that exploits the discrete nature of the object by alternating iteratively between discretization steps of segmentation based on gray values, and continuous steps of reconstruction on the boundary of the segmented image. DART has been successfully used for reconstructing samples from applications in CT and ET.

Despite its superior performance demonstrated in some cases, applying DART in practice is still a challenging and time consuming process. This difficulty is related to requires many parameters to be specified by the user. Figuring out the optimal choices for these parameters for experimental data requires substantial effort and time for manual tuning. Second, one of the important concepts behind dart is to reduce the number of unknowns by fixing and removing the pixels/voxels within the flat regions of the segmented image from the system of linear equations. This makes the system better determined as the reconstruction improves through iterations. However, fixing the interior regions of the segmented image can misassign a substantial number of pixels in the reconstruction especially when the reconstruction is still not accurate in the early stage. This can push the solution in the wrong direction when the limited projection data also contains a certain mismatch from the model (e.g., image alignment error, nonlinearity in image formation) and/or moderate to high level of noise. Third, dart applies a smoothing filter to the free pixels as a way to even out the fluctuations over the boundary pixels and combat the influence of noise in the projection data. Although this strategy makes sense intuitively, it is hard to predict its effect on the reconstruction under noisy conditions. Last but not least, the reconstruction can be not entirely discrete in practice due to imperfection of the imaging system or the object itself. The hard segmentation in the discrete step of DART imposes a strong constraint on the solution, making it difficult to cope with practical complications.

Efforts have been made to deal with some of the above mentioned problems. For example, an algorithm was proposed that couples DART with a search-based algorithm in order to find the right gray values for the reconstruction. An algorithm known as SDART was proposed to deal with noisy projections by spreading the noise across the wholeimage domain using a penalty matrix.

In this paper, we propose a new iterative reconstruction algorithm, TVR-DART, which can produce more accurate reconstructions than DART under noisy conditions from limited projection data. TVR-DART takes the key concept of DART in terms of steering the solution toward discrete gray values, and incorporates this strategy within an automated optimization framework of compressive sensing. We replace the hard segmentation step of DART with a soft segmentation function which is described with a sum of logistic functions. This smoothes the objective function and allows us to solve the discrete reconstruction and gray value estimation problems alternatingly in a non-convex optimization framework. A total variation term applied on the segmented reconstruction is added to the objective function to combat noise and regularize the reconstruction under extremely limited data conditions.The core idea of TVR-DART is to gently push the gray values of the reconstruction using the soft segmentation function ateach iteration, and to continuously update the entire image so that the reconstruction after applying the soft segmentation better matches the projection data and is at the same time sparse in its boundaries between regions with constant discrete gray values. Due to the fact that the _1-norm is applied on the segmented reconstruction, TVR-DART produces accurate and sharp boundaries in the reconstruction without blurring

II. Mathematical Concepts and Dart Algorithm

A. Problem Definition

We treat the tomographic reconstruction problem as a system of linear equations. The formulation is generic and covers 2D and 3D imaging in parallel/fan/cone beam geometries. Let $\boldsymbol{x} = \_x\boldsymbol{j}\_ \in \mathrm{R}n$ denote a vector containing the discretized pixel/voxel values of the object being imaged, while $\boldsymbol{p} = (p\boldsymbol{i}) \in \mathrm{R}m$ represents the measured projection values from all detectors collapsed into a single vector. Then the forward image formation process can be modeled using a projection operator $\boldsymbol{W} = \_w\boldsymbol{ij}\_ \in \mathrm{R}m\mathrm{x}n$ which maps the object to the measured projection data:

$$\boldsymbol{p} = \boldsymbol{Wx} \ (1)$$

In discrete tomography, we intend to solve the following problem: Let G > 1 be the number of gray values and $R = \{\rho 1, \ldots, \rho\mathrm{G}\}$ denote the set of gray values. Find $\boldsymbol{x} \in Rn$ such that $\boldsymbol{Wx} = \boldsymbol{p}$. In practice a solution can be seen as the result of the following optimization:

Min $x \in \{\rho 1, \ldots, \rho \mathrm{G}\}n$

$|| \boldsymbol{Wx} - \boldsymbol{p} ||$ (2)

where _• _ represents a certain norm. Notice that because the solution space Rn is not a convex set, many algorithms from convex optimization cannot be directly applied for the reconstruction problem in discrete tomography.

B. Original DART Algorithm

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Here we provide a brief overview of the DART algorithm. More details can be found in [13]. DART alternates between a continuous algebraic reconstruction step and a discrete segmentation step, and uses a well-designed procedure to gradually improve the segmented image.

The algorithm starts with an initial continuous reconstruction using an algebraic reconstruction method (ARM). As the first step, a hard segmentation is applied on the reconstruction, rounding all pixels to the nearest gray value in *R*. In the second step, *boundary* pixels, that have at least one neighbour pixel with a different gray value, and some randomly chosen pixels from the non-boundary region are selected as *free* pixels. The probability of a *non-boundary* pixel to be classified as *free* pixel is $1 - p$, with *p* known as the *fix probability*. In the third step, the unknowns corresponding to the pixels other than *free* pixels, known as *fixed* pixels, are removed from the system of linear equations in (1). Specifically the corresponding columns in ***W*** and rows in ***x*** are removed while the projections of the *fixed* pixels are subtracted from ***p***. Then the *free* pixels are updated with a continuous reconstruction step with the ARM. In the fourth step, the free pixels are smoothed with a Gaussian filter with a small kernel as a way to regulate strong fluctuations. The whole procedure iterates between step 1 and 4 until a termination criterion is met.

III. Formulations for TVR-Dart

In TVR-DART we combine the concept of solution steering of DART with TV regularization, and solve the discrete tomography problem within an automated optimization framework. The objective function *F* consists of two parts: a data fit term *F*fit incorporating the discrete prior and a regularization term *F*reg ensuring the sparsity of image gradients: where λ is the weight for controlling the trade-off betweenthe two parts of the objective function. *S* _*x*, *R*_ represents the Soft Segmentation Function that smoothly pushes the gray values toward discrete solutions, and *M*_ (•) is the Huber norm function which interpolates between smooth _2 treatment of small residuals and robust _1 treatment of large residuals with _ as the threshold between the two types of norms. *R* is the set of gray values augmented with the set of thresholds $\{\tau 1, \ldots, \tau G\}$,is the spatial image representation of vector ***x***, and ∇ represents the discrete gradient operator. The inclusion of the soft segmentation in the objective function applies a soft push on the pixel values that encourages discrete solutions. The Huber norm of the image gradient is applied to steer towards sparse solutions and it yields a differentiable objective function. TVR-DART aims to minimize the objective function (3) over both the reconstruction ***x*** and the soft segmentation parameters *R*: where G denotes the prior knowledge in terms of the totalnumber of discrete gray values in the reconstruction.

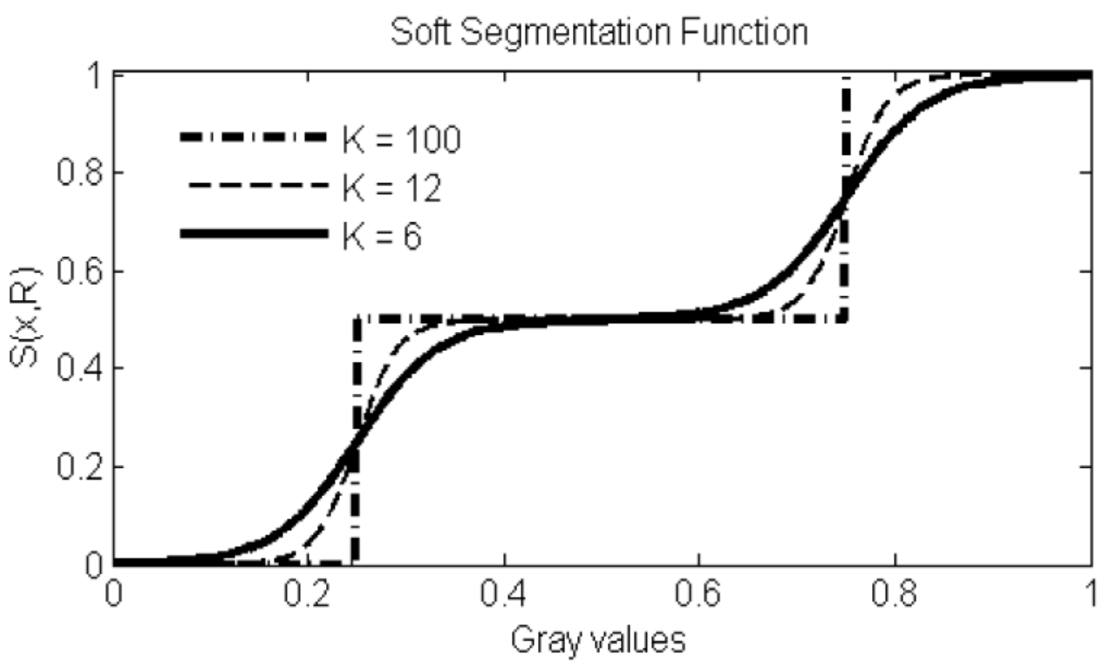

Fig. 1. Illustration of the soft segmentation function *S (x, R)* under different values of *K*. The gray values are (0, 0.5, 1.0) with thresholds at (0.25, 0.75).

The objective function in (3)-(5) means that TVR-DART pursues a reconstruction that matches well with the projection data in a least square sense after applying the segmentation function, and the segmented solution is preferred to also exhibit sparse boundaries. Therefore, the solution we are looking for in the end is not ***x*** but *S* _*x*, *R*_ which is optimized to fit with the projection data. By changing the formulation of discrete tomography from (2) to (7), we manage to switch the problem of a convex objective function on

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a non-convex set, to a problem of a non-convex objective function over a convex set, which can be solved using non-convex optimization techniques. Previous attempts of limited data discrete tomography using non-convex optimization techniques can be found. In CT, non-convex prior models were also proposed for reconstruction from nonlinear X-ray measurement..

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